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# The $\text{Si}_{1-x}\text{N}_x$ Rugate

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<p>The effectiveness of a Rugate filter constructed of a silicon-nitrogen alloy is calculated. Single and double line rugates are discussed. The dependence on the rugate on the various parameters such as the number of cycles, the amplitudes of the sine waves and the phase angle are presented. The effects of absorption, constructional malfunctions, and the angle of incidence of the laser beam are studied. It is found that quite large optical densities can be achieved with rather thin films, thus the silicon-nitrogen rugate is a viable candidate for a laser rejection filter.</p>						
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## THE $\text{Si}_{1-x}\text{N}_x$ RUGATE

### Section I: Introduction

With the advent of lasers, intense sources of monochromatic light are now available. The problem of eye and sensor protection from these devices is now crucial to the field operations of troops and military weapons systems. In other words it is necessary to construct a filter which will reject particular laser lines from impinging upon a material. One possible solution, if only one line is to be rejected, is to have a filter made up of a superposition of dielectric layers (dielectric stacks). This solution suffers from the fact that these stacks suffer an abrupt change of composition at the interfaces which could lead to large stresses, increased absorption and differential thermal expansion. These problems are avoided in the concept of the rugate because very slow changes in material are inherent in the rugate, thus obviating the problems inherent in the use of dielectric layers. In addition more than one line can be rejected with the use of the rugate. This article describes calculations of a silicon-nitrogen alloy rugate material. We shall show that quite large effects can be had with rather thin filters, if reasonable limits on constructional errors are maintained.

The present article presents results in the infra-red and near infra-red region of the spectrum. Preliminary calculations of rugates for the visible wavelengths have been performed and will be extended and presented in a future report

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## Section II: Theory

The concept of the rugate was first suggested by Phillippi and Davidson<sup>(1)</sup>. This idea was to have a number of quarter-wave plates with given reflectance  $R$  add coherently to give total reflectance for a pre-assigned wavelength. The formula proposed by the above authors was to vary the index of refraction sinusoidally according to

$$\hat{n}(x) = \hat{n}_a(x) + \sum_{i=1}^m \frac{\Delta n_i}{2} \sin\left(\frac{2\pi x}{P_i} + \delta\right) \quad (1)$$

Here we have allowed for the fact that the superposition of sines allows the rugate to reject  $m$  lines (in principle). The hat above each symbol means that the quantity is complex. Here  $n_a$  is the average value of the index of refraction,  $\Delta n_i/2$  is the partial amplitude of the sinusoidally varying part and  $x$  is the radial value of the extent of the rugate. The period of each sine wave is given by  $\lambda_i = 2n_a P_i$ , where  $\lambda_i$  is the wavelength of the line rejected, and  $P_i$  is twice the thickness of an equivalent quarter wave filter.

The average index of refraction, is in part, a function of wavelength and its real part is given quite accurately by the Herzberger dispersion formula<sup>(2)</sup>

$$n = A + BL + CL^2 + D\lambda^2 + E\lambda^4 \text{ where } L = 1/(\lambda^2 - 0.028), \text{ with the wavelength in microns.}$$

The coefficients are  $A = 3.41906$ ,  $B = 1.23172 \times 10^{-1}$ ,  $C = 2.65465 \times 10^{-2}$ ,  $D = -2.66511 \times 10^{-8}$ , and  $E = 5.45852 \times 10^{-14}$  for Si. This formula describes the real part of the index of refraction of silicon from 1.12 to 588 microns. Below that wavelength no dispersion formula holds, so we extrapolate. No such simple dispersion relation holds for the  $\Delta n_i$ . Since  $n$  within the rugate is a complicated function of  $Si_{1-x}N_x$ , the dispersion formula is only used as a boundary condition.

The trick then is to choose some average value of  $n_a$  which comes from a superposition of nitrogen and silicon and then choose  $\Delta n_i$  to satisfy the criteria<sup>(1)</sup>;

$$D_i = C (\Delta n_i N) / n_a + C_2, \quad (2)$$

where  $N$  the number of cycles and  $D$  is the optical density. Here  $C_2$  allows for the fact that for most alloys the transmittance is non zero, but not unity, in the limit  $\Delta n_i$  goes to zero. The transmission is then given by

$$T_i = 10^{-D_i}. \quad (3)$$

The rejection band is not infinitely sharp at the central wavelength but has a finite half width proportional to

$$\lambda_i \Delta n_i / n_a. \quad (4)$$

A balance must be struck by choosing the number of cycles  $N$  so that the resulting rugate is structurally sound and by choosing  $\Delta n_i$  to compromise between a maximum optical density and a minimum width. The maximum  $\sum_i \Delta n_i = 3.9$  for the NRL rugate.

We should emphasize that the index of refraction is complex and details of the imaginary part depend critically on the wavelength. No simple dispersion formula can be found. In general we write  $\hat{n}_a = n_a + ik$ , and  $\hat{\Delta n}_i = \Delta n_i + i \Delta k$ .

Section III: General results for a rugate to reject the 2.8 micron line of the HF laser.

We have modified an existing program to calculate  $R$  and  $T$  for multi-layers to be replaced by sinusoidally varying parts of the index of refraction. We normalize the incident and outgoing flux by imposing  $R+T+A=1$ . Here  $R$  is the reflectance,  $T$  the transmittance and  $A$  is the amount of flux taken out of the beam by absorption (imaginary index of refraction).

Figure 1 shows the dependence of the reflectance on the phase shift  $\delta$ , (Essentially the boundary condition of the index of refraction of the rugate as the beam enters from air). It is to be noted that phase shifts of  $0^\circ$  and  $180^\circ$  give essentially the same pattern, while a  $90^\circ$  phase shift lowers the reflectance for wavelengths greater than 2.8 microns and raises the reflectance for wavelengths between the 2.8 micron stop band and a secondary (first harmonic) maximum at 1.4 microns. A phase shift of  $270^\circ$  reverses the pattern from that of  $\delta=90^\circ$ .

Figure 2 demonstrates the dependence of  $-\text{LOG}_{10}(T)$  on  $\Delta n$  and fixed  $N=20$  as we decrease  $\Delta n$  from 1.0 to 0.6 and 0.2. The transmittance passes from  $10^{-9}$  to  $10^{-11/2}$  to  $10^{-1.8}$  at the peak wavelength of 2.8 microns.

Figure 3 shows the dependence of  $-\text{LOG}_{10}(T)$  on the number of cycles  $N$  of the rugate for fixed  $\Delta n$  of 1. Increasing the number of cycles certainly improves the optical density of the filter, with a concomitant increase in film thickness  $L=NP$ .

Figure 4 demonstrate the effects of absorption. The choice of  $k$  for this calculation was taken from amorphous silicon nitride  $(\text{Si}_3\text{N}_4)^{(2)}$ . As shown in reference 2,  $k$  is a decreasing function of wavelength. For a wavelength of 2.8 microns no data exists. However, we have arbitrarily chosen  $k=2.2 \times 10^{-4}$ , its value at 0.2755 microns. This is certainly too large, for example  $k$  for Si at  $\lambda=2.7144$  microns is only  $2.5 \times 10^{-12}^{(2)}$ , but we use it to demonstrate the effect. We take the number of cycles  $N=20$ , the average index as  $n_a=3$  and  $\Delta n=1$ .

As can be seen in figure 4, there is an absorption maximum and minimum at the first harmonic ( $\lambda=1.4$  microns). At the fundamental wavelength, 2.8 microns, the absorption is practically zero, bounded by rather sharp peaks at 2.6 and 3.05 microns. We do not understand these maxima because there is no structure in amorphous silicon nitride which would explain these features. In any case our

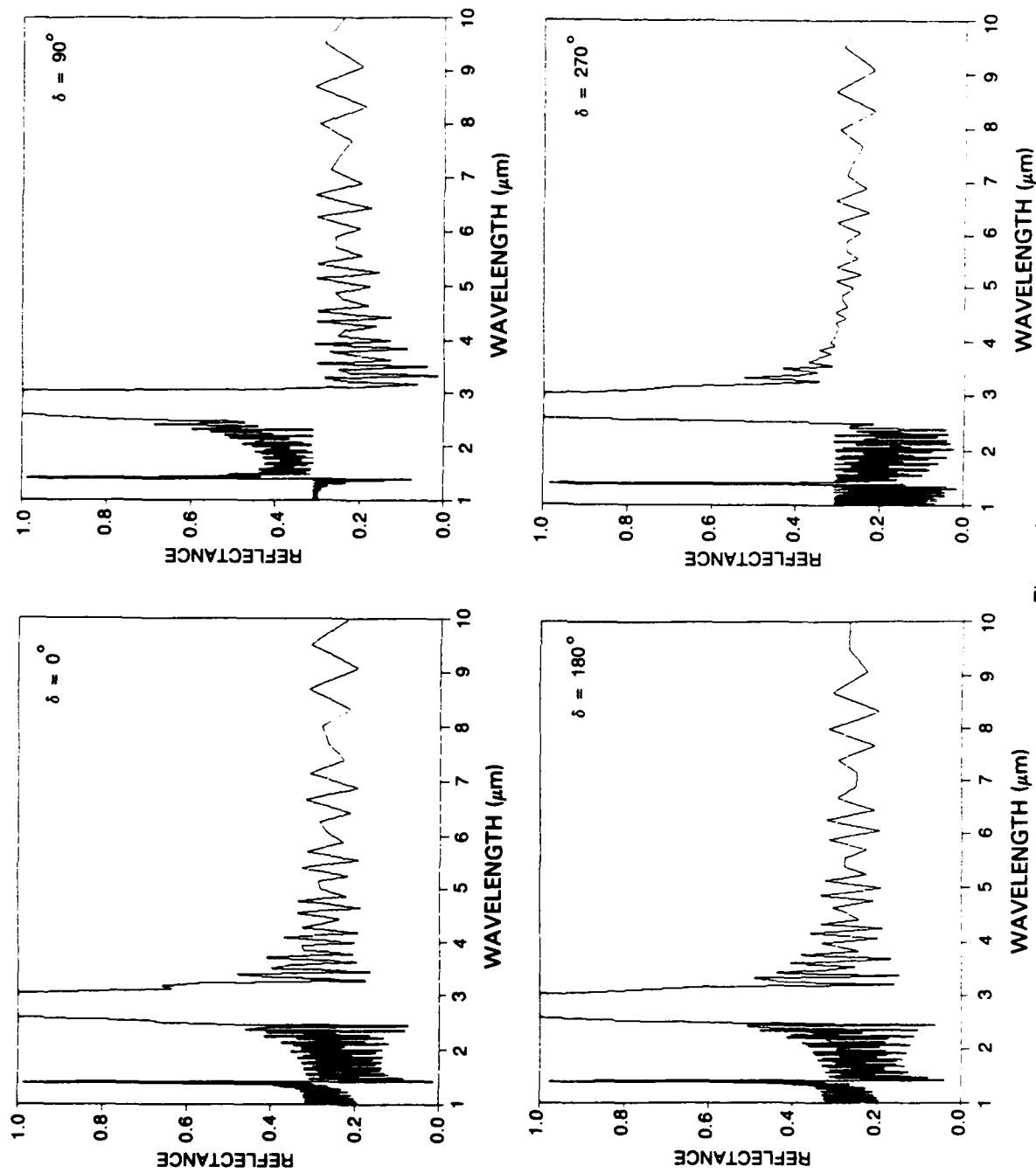


Figure 1

REFLECTIVITY SPECTRA AS A FUNCTION OF PHASE FOR  $\lambda = 2.8 \mu\text{m}$   $N = 10$   $n_a = 3$   $\Delta n = 1$ ,  $P = .4667 \mu\text{m}$

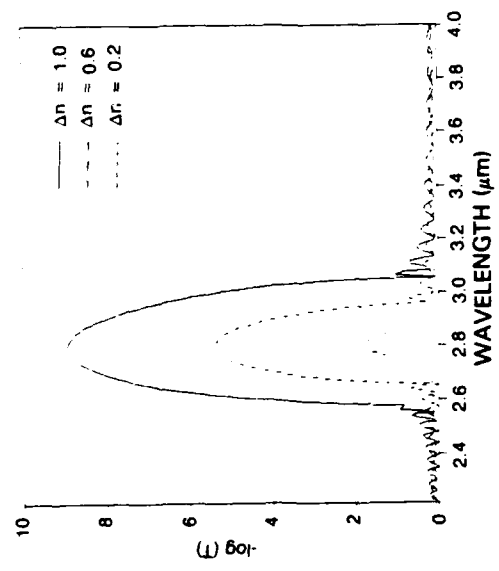


Figure 2  
THE NEGATIVE OF Log T AS A FUNCTION OF THE  
AMPLITUDE  $\Delta n$ . FOR  $\lambda_0 = 2.8 \mu m$   
 $N = 20$   $n_a = 3$   $P = .4667$

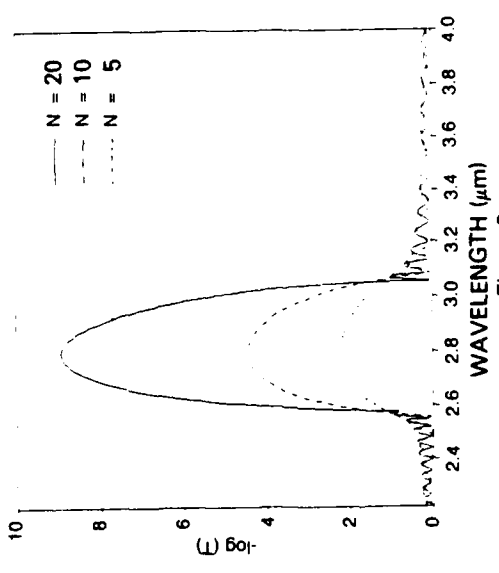
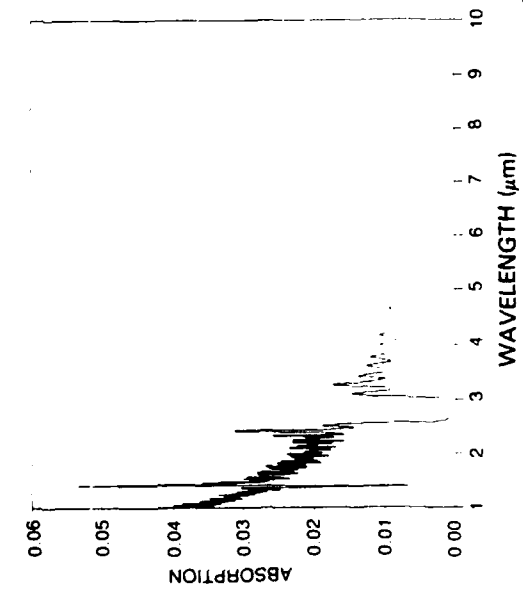


Figure 3  
THE NEGATIVE OF Log T AS A FUNCTION OF THE  
NUMBER OF CYCLES FOR  $\lambda_0 = 2.8 \mu m$



a. ABSORPTION SPECTRA OF RUGATE  $1 \leq \lambda \leq 10 \mu m$

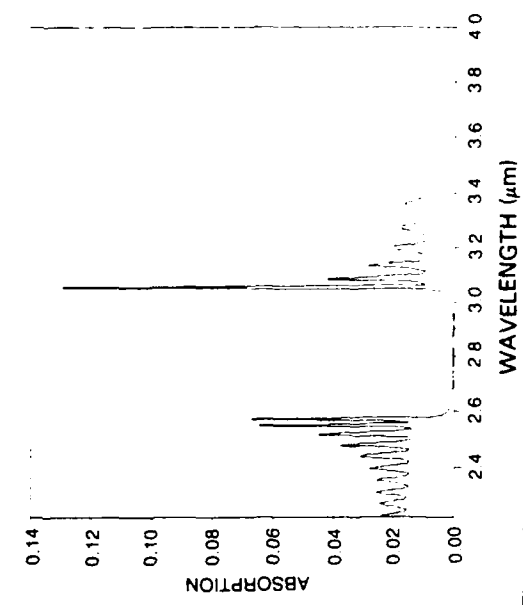


Figure 4  
b. DETAIL OF ABSORPTION SPECTRUM NEAR  
TRANSMISSION MINIMUM.  $N = 20$ ,  $n_a = 3$ ,  
 $\Delta n = 1$ ,  $P = .4667 \mu m$

estimates are probably much too large.

In practical cases it is necessary to allow for the machinery to malfunction, giving unwanted experimental fluctuations in the presumably perfect sine wave.

We have performed two types of calculations. The first was to allow random fluctuations in the sine wave occurring quite frequently. No appreciable effect was found in the performance of the rugate. No effect could be shown on a graph.

The second was to allow systematic fluctuations in optical length (the product of  $n$  and  $Z$ , the step length). The first case we tried was to allow a systematic uncertainty bounded by 10%. As can be seen in figure 5 the peak of the optical density shifts and increases, while large secondary reflectances occur.

In figure 6 are shown the optical density and reflection spectra when the systematic fluctuation is held to within 2.5%. Essentially there is very little effect. The peaks of the optical density and reflection spectrum do not shift appreciably, and the secondary maxima of the reflectance are much decreased. The result in figures 5 and 6 imply that up to 3% errors in construction of the rugate can be tolerated.

In figure 7 are plotted the results when the laser beam is not incident from the normal. Again we have used  $N=20$ ,  $n_a=3$  and  $\Delta n=1$ . At  $15^\circ$  the optical density is still quite large.  $30^\circ$  incidence still provides an optical density of 6 for the s wave and 6.5 for the p wave. Only when the angle of incidence is of order  $45^\circ$  does the optical density essentially vanish. We attribute these results to the large average index of refraction,  $n_a$ , and the large  $\Delta n$  which bends the light toward the normal and provide a relatively large halfwidth of the rejection band, respectively.

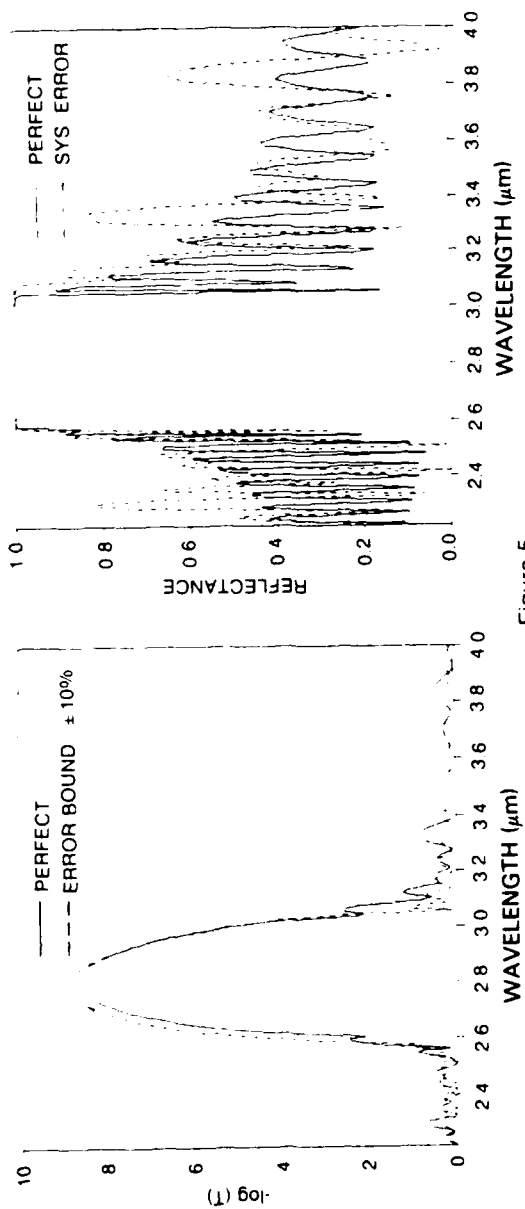


Figure 5

a. EFFECT OF SYSTEMATIC ERROR IN OPTICAL LENGTH. ERROR BOUNDS  $\pm 10\%$

b. REFLECTION SPECTRA  
 $N = 20, n_a = 3, \Delta n = 1,$   
 $P = .4667 (\mu\text{m})$

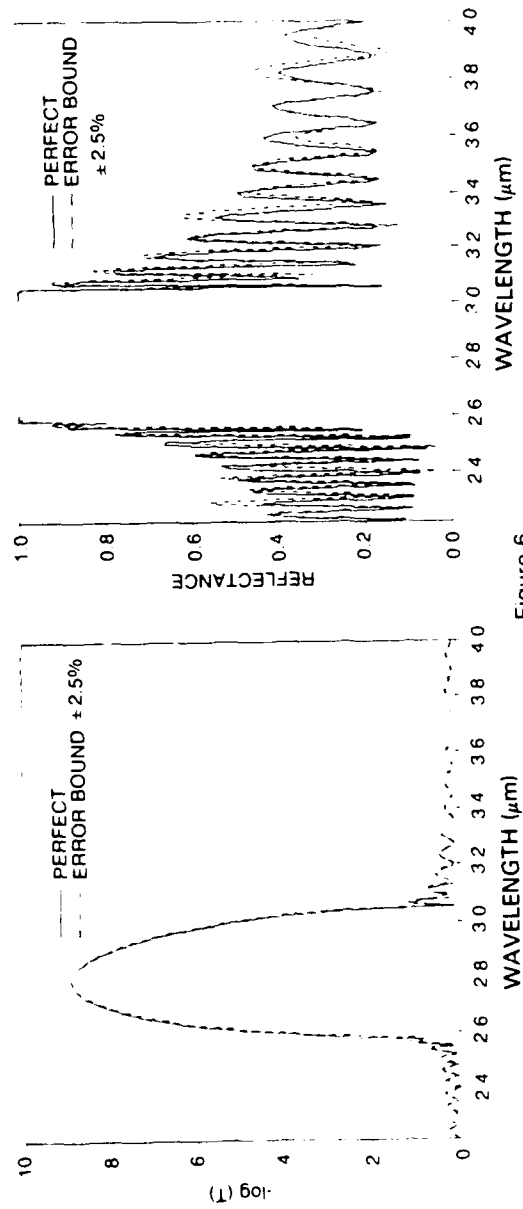


Figure 6

SAME AS Figure 5 EXCEPT SYSTEMATIC ERROR BOUNDS CONSTRAINED TO  $\pm 2.5\%$

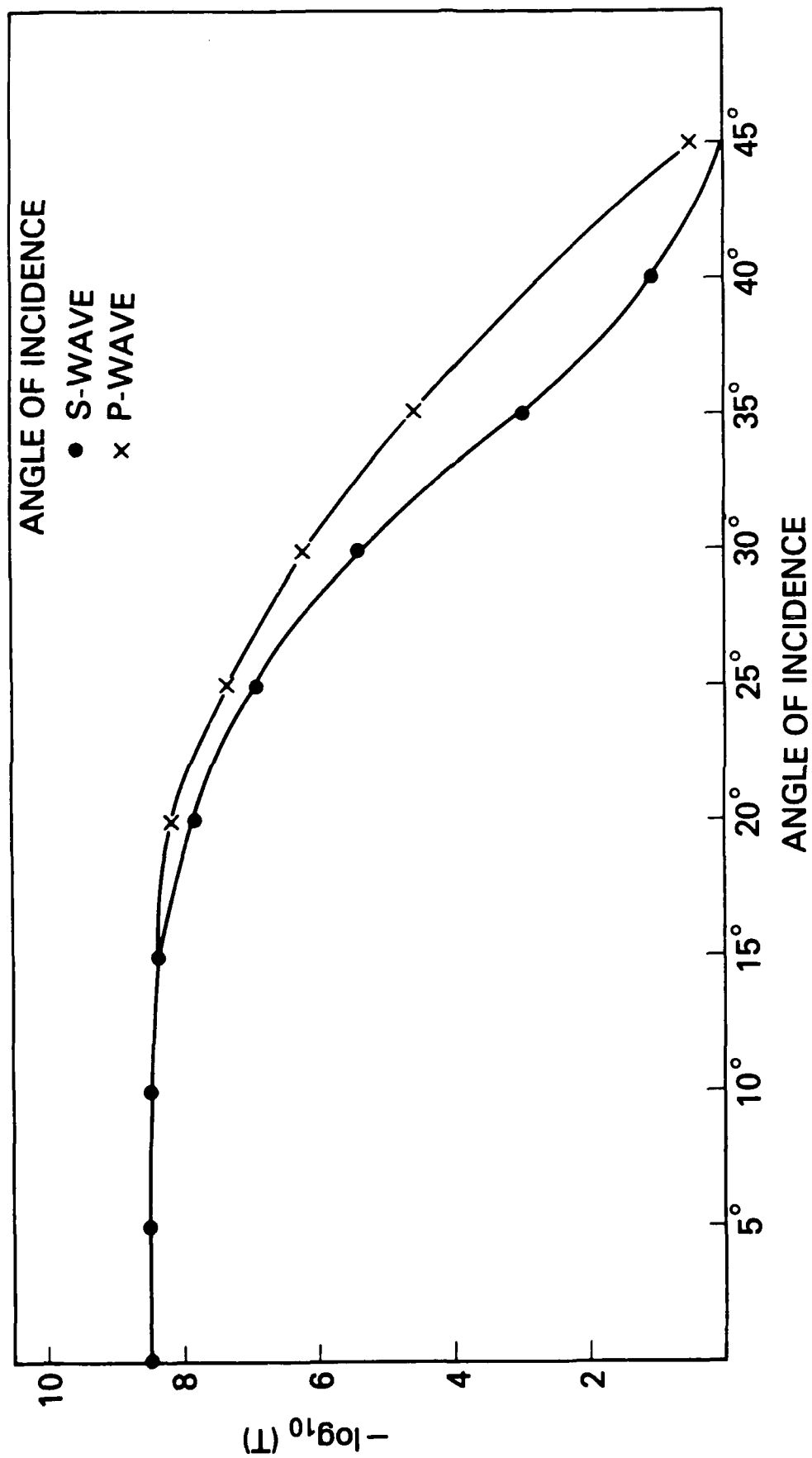


Figure 7 Dependence of the optical density on angle of incidence.

#### Section IV: Results For Two Line Rugates

In this section we describe the construction of a two line rugate. We choose to eliminate the 2.8 micron HF line and the 1.06 micron line for the Neodymium YAG lasers. Figure 8 shows the optical density for these two lines. The parameters are  $N=10$ ,  $n_a=3$ , and  $\Delta n=.4$  for each line. We choose  $N$  to represent the number of cycles for the 2.8 line, i.e. that with the longest period. Note that 1.06  $\mu\text{m}$  line, the line with the shorter period, sees effectively many more cycles and is much less strongly transmitted. Note also that the width for the 1.06 line is much smaller, as predicted, since the width is proportional to  $\lambda_1$ . Figure 9 shows the reflectance spectra for the case described above. Both the 2.8 micron line and the 1.06 micron line are strongly reflected. Note the presence of a first harmonic at 1.4 microns. Although not strong, the rugate definitely shows satellite lines at the harmonics of the fundamental.

The same general remarks hold for the two line rugate as for the one line rugate. We have chosen to illustrate only  $\delta=0^\circ$ . Doubling  $N$  or  $\Delta n$  will increase the optical density and/or the width according to equations 2-4 in Section I.

#### Section V: Possible Application

It has been suggested by C. A. Carosella that if a multi-line rugate could be designed so that the band between 3 microns and 5 microns could be totally rejected, that this could have importance in the protection of military IR sensor systems. Figure 10 shows an attempt to bridge this gap with two lines using the maximum  $\Delta n$ , of 1.95 for each line. As can be seen there is a gap centered at 3.9 microns which is not totally reflecting.

Shown in figure 11 is an attempt to eliminate this gap using three lines, again using maximum  $\Delta n$  of 1.3, to provide the necessary width. This shows gaps at 3.5 microns; 4.1 microns and a fall off at 4.75 microns. Neither combination

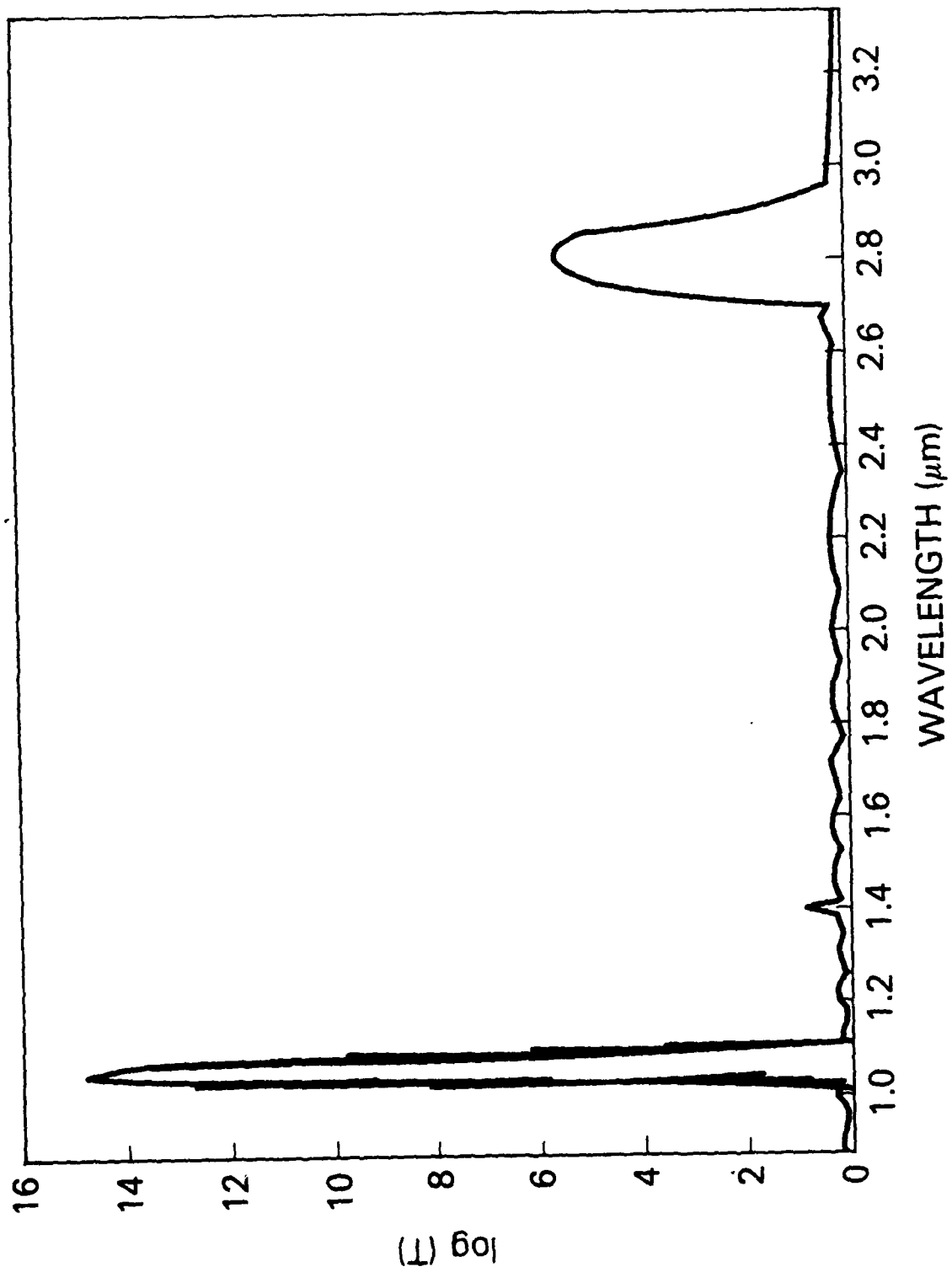


Figure 8 Optical Density for the two line rugate.

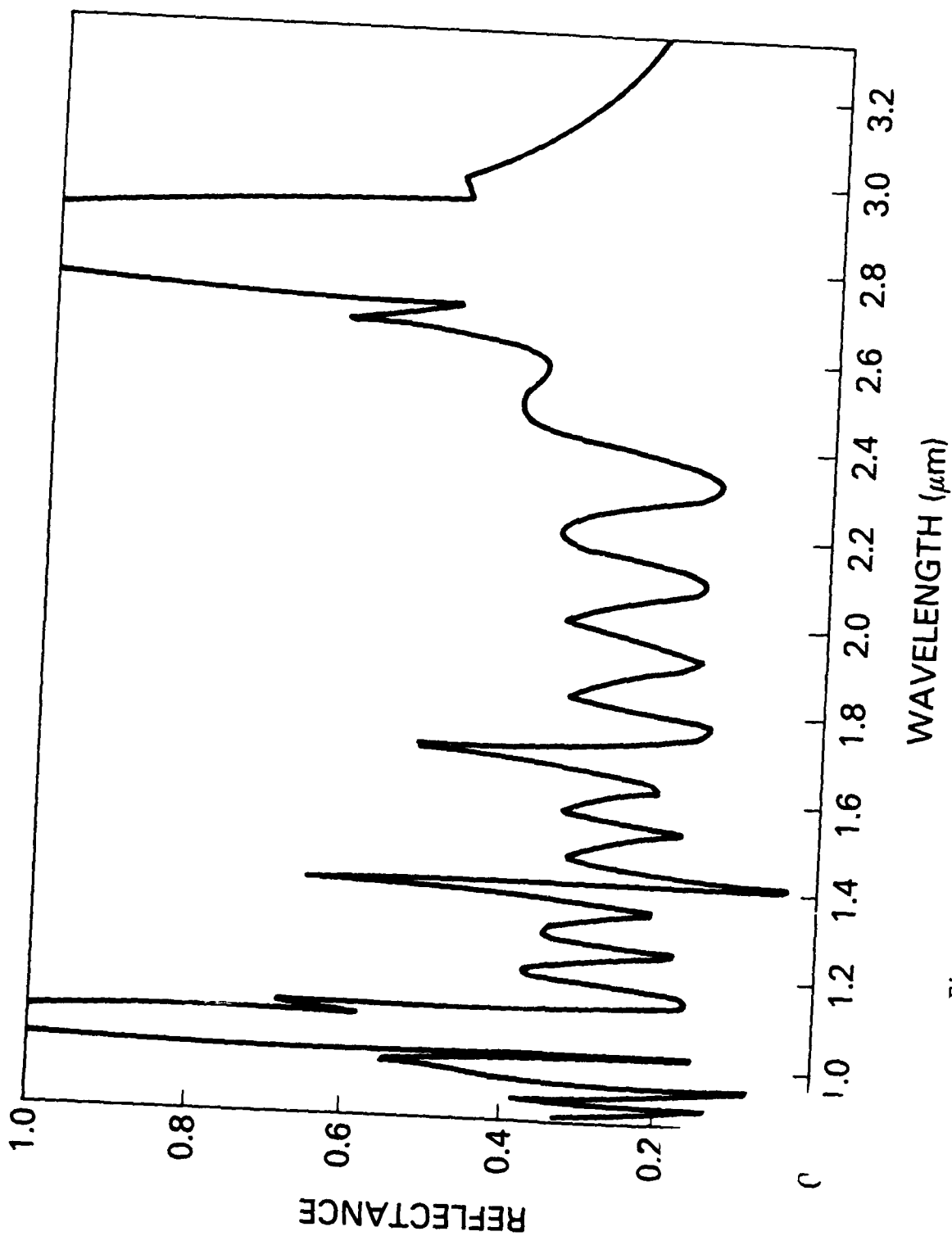


Figure 9 Reflectance Spectrum for the two line rugate.

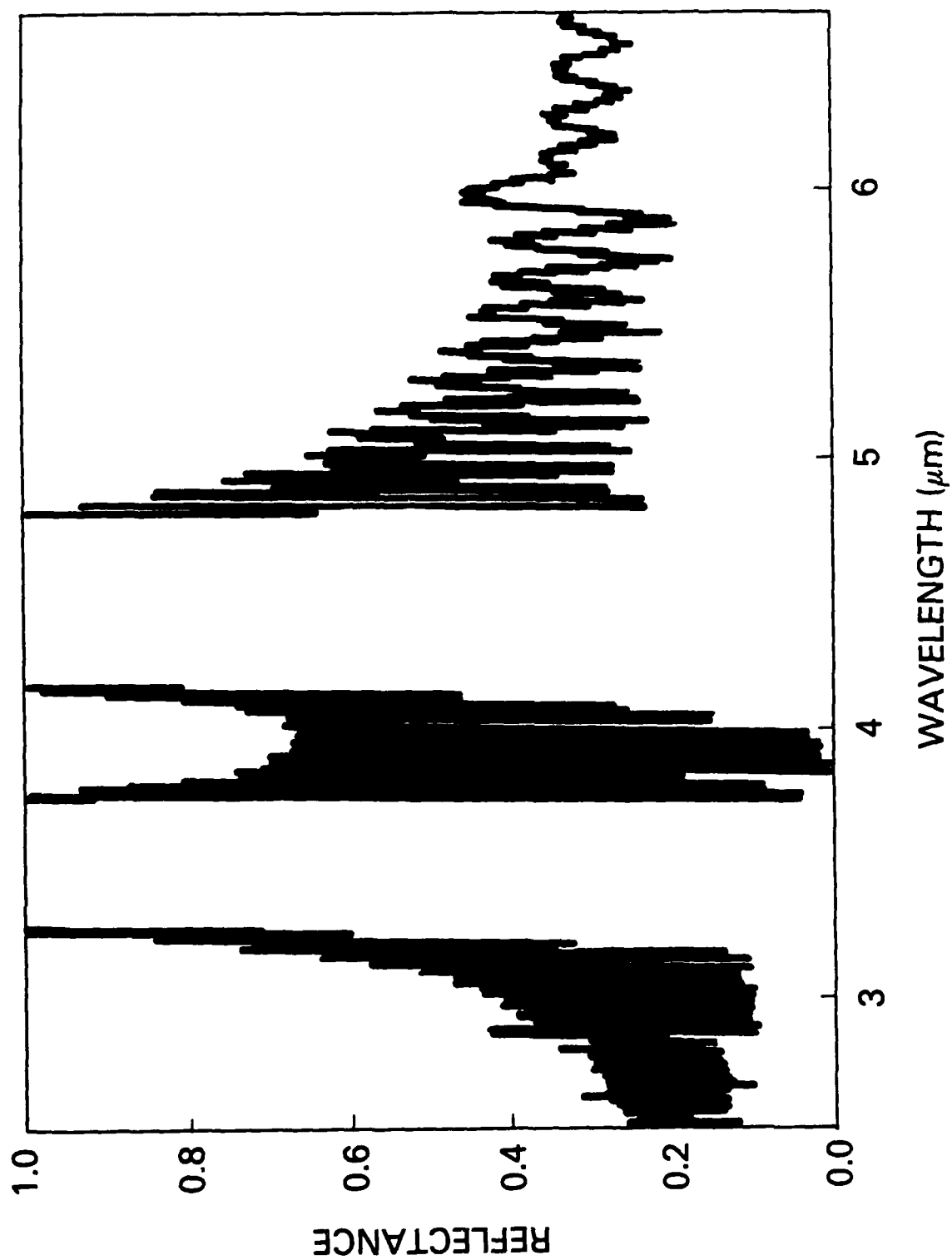


Figure 10 Attempt to bridge 3-5 micron gap with two lines.

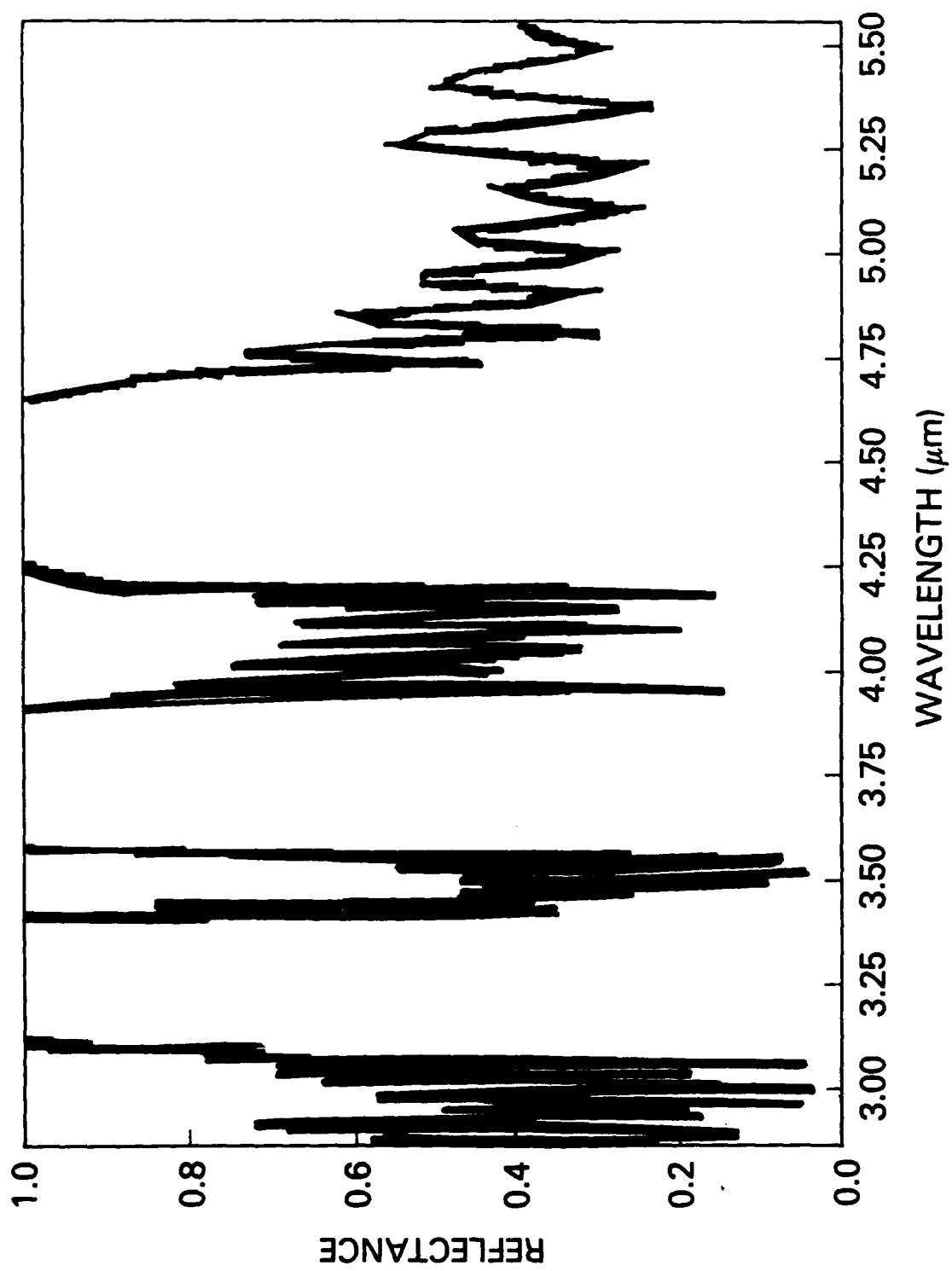


Figure 11 Attempt to bridge 3-5 micron gap with three lines.

is able to give total reflection for the 3 micron to 5 micron band, although the two line rugate is perhaps more successful.

#### Section VI: Summary and Conclusions

Our results clearly demonstrate the success of the silicon-nitrogen rugate in the infra-red region. Quite thin films of the order of 5 microns produce an optical density of 5 for the 2.8  $\mu\text{m}$  line of the HF Laser. Another filter which superimposes two sines gives equally gratifying results. Because of the large index of refraction and the large band width, the rugate gives ample reflectance for angles of incidence less than 30°. Although our demonstration used large values of  $N$  and  $\Delta n$ , more moderate values could be chosen to decrease the width of the stop band, with the penalty of requiring thicker films for the same optical density. Finally, we should emphasize that systematic errors should be held to less than 3%, while random fluctuations play no role.

To our knowledge the results for constructional errors, non normal incidence of laser light impinging on the rugate, the effects of realistic absorption, and the presence of strong harmonics has heretofore not been presented.

#### Section VII: Acknowledgements

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Thanks are also due to G.H. Herling for his advice on a few vexing VAX questions.

#### Section VIII: References

1. "Rugate Filters - A New Approach to Laser Hardened Optics", C.M. Phillippi and J.F. Davison, Proceedings of the Fifth DOD Conference on Laser Vulnerability Effects and Hardening, 1983
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